Improving Core Resilience of Network under Random Edge Deletion

Ricky Laishram, Syracuse University, rlaishra@syr.edu

Ahmet Erdem Sarıyüce, University at Buffalo, erdem@buffalo.edu

Tina Eliassi-Rad, Northeastern University, eliassi@neu.edu

Ali Pinar, Sandia National Laboratories, apinar@sandia.gov

Sucheta Soundarajan, Syracuse University, susounda@syr.edu

Introduction

- The *k*-core of a network is the maximal sub-graph in which every node has at least k neighbors [1].
- Nodes that belong to higher *k*-cores are considered to be more central within the network.
- The idea of *k*-cores have been used in to various applications – studying the internet [2], predicting protein functions [3] etc.

Problem

- How can we characterize the resilience of the *k*-core structure of a network against random edge deletion?
- Given a network, how can we improve the the resilience of its *k*-core against edge deletion?

Proposed Measures

Network Level Property:

- ***n,p*-Core Resilience**: It is defined as the rank correlation between the ranking of the top *n%* nodes (ranked by core number) in the original network to that of the network after *p%* edges have been removed uniformly at random.

Node Level Property:

- **Core Strength**: It is a measure of how robust the node's core number is against edge deletion.

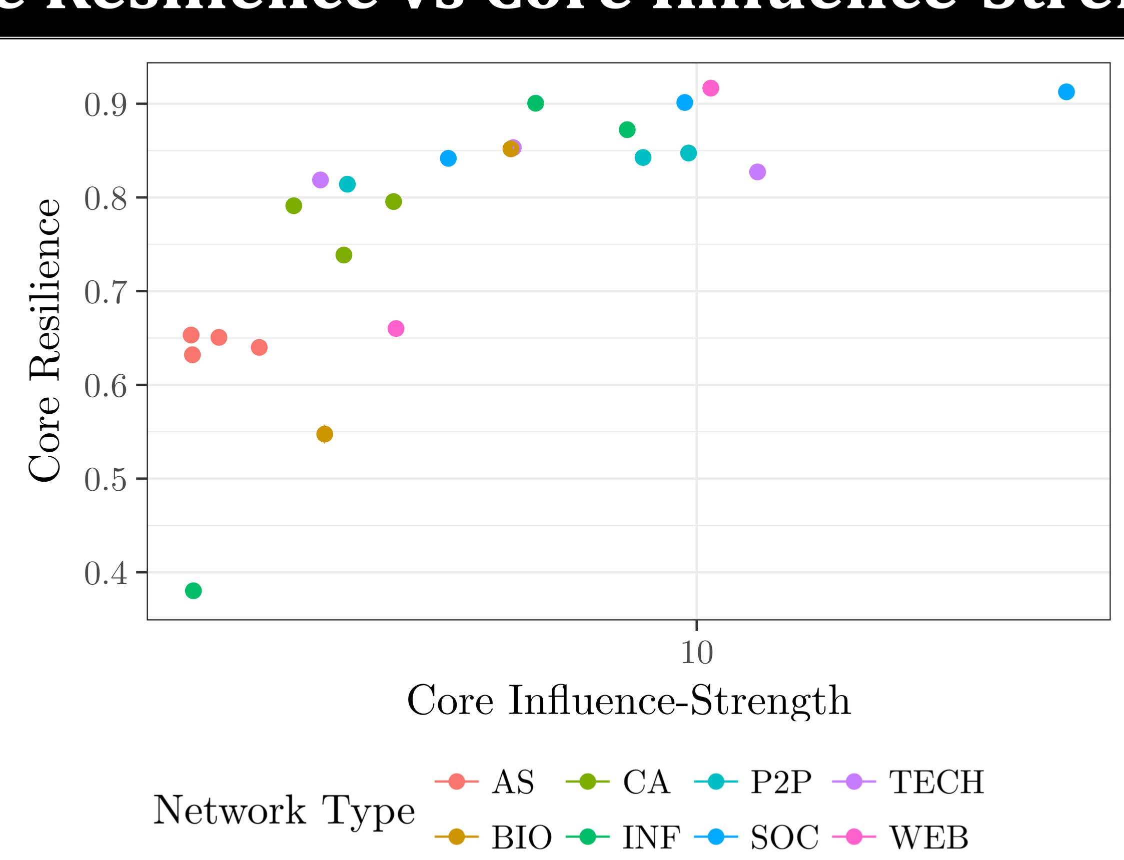
$$CS(u,G) = |\{v : v \in \Gamma(u,G), K(u,G) \leq K(v,G)\}| - K(u,G)$$

- **Core Influence**: It is a measure of how much other nodes relies on it for their core numbers

$$CI(u,G) = \sum_{v \in V_\delta \cap \Delta_<(u,G)} CI(v,G) \Big/ |\Delta_>(v,G)|$$

Networks in which the nodes with high Core Influence also have high Core Strength are also likely to have high Core Resilience.

Core Resilience vs Core Influence-Strength



Full Paper

Ricky Laishram, Ahmet Erdem Sarıyüce, Tina Eliassi-Rad, Ali Pinar, and Sucheta Soundarajan. Measuring and Improving the Core Resilience of Networks. *In WWW 2018: The 2018 Web Conference, April 23–27, 2018, Lyon, France.*

Improving Core Resilience

Problem: Given a graph G and edge budget b, which b edges should we add so that the core resilience of the modified network is as high as possible, without changing the core numbers?

Idea: Add the edges in such a way that it improves the Core Strength of the nodes with high Core Influence.

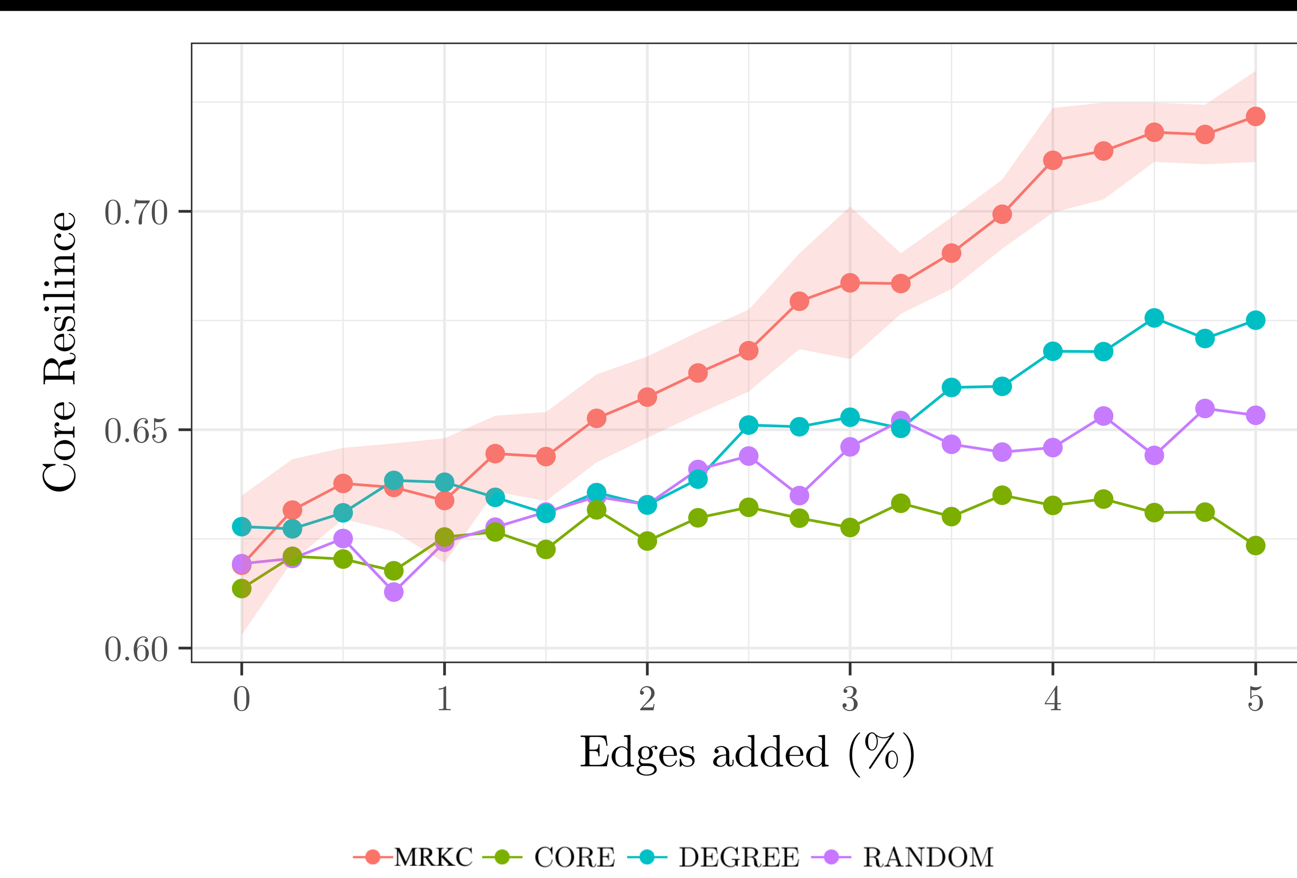
Maximize Resilience of k-Core (MRKC)

- Let E' be the set of edges that do not exist in G.
- Filter out edges from E' that would change the core number if they are added to G [4].
- For all $(u,v) \in E'$, assign edge priority as,

$$\rho(u,v) = \begin{cases} \frac{CI(u,G)}{CS(u,G)} & \text{if } K(u,G) < K(v,G) \\ \frac{CI(v,G)}{CS(v,G)} & \text{if } K(u,G) > K(v,G) \\ \frac{CI(u,G)}{CS(u,G)} + \frac{CI(v,G)}{CS(v,G)} & \text{if } K(u,G) = K(v,G) \end{cases}$$

- Add the edges in decreasing order of the priority.

Result



Notations

$\Gamma(u,G)$	Neighbors of node *u* in *G*
$K(u,G)$	Core number of node *u* in *G*
$\Delta_>(u,G)$	Neighbors of *u* with core numbers greater than that of *u* in *G*

References

1. Stephen B Seidman. 1983. Network structure and minimum degree. Social networks 5, 3 (1983), 269–287.
2. Shai Carmi, Shlomo Havlin, Scott Kirkpatrick, Yuval Shavitt, and EranShir. 2007. A model of Internet topology using k-shell decomposition. Proceedings of the National Academy of Sciences 104, 27 (2007), 11150–11154.
3. Md Altaf-Ul-Amine, Kensaku Nishikata, Toshihiro Korna, Teppei Miyasato, Yoko Shinbo, Md Arifuzzaman, Chieko Wada, Maki Maeda, Taku Oshima, Hirotada Mori, et al. 2003. Prediction of protein functions based on k-cores of protein-protein interaction networks and amino acid sequences. Genome Informatics 14 (2003), 498–499.
4. Ahmet Erdem Sarıyüce, Buğra Gedik, Gabriela Jacques-Silva, Kun-Lung Wu, and Ü. V. Çatalyürek. 2013. Streaming Algorithms for K-core Decomposition. Proc. VLDB Endow.6,6(April2013), 433–444.